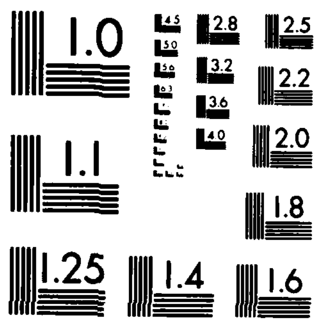
AD-A149 817 AXIOMATIC CHARACTERIZATIONS OF CONTINUUM STRUCTURE FUNCTIONS(U) STATE UNIV OF NEW YORK AT STONY BROOK DEPT OF APPLIED MATHEMA.. C KIM ET AL. NOV 84 1/1

UNCLASSIFIED AFOSR-TR-84-1202 AFOSR-84-0243 F/G 20/11 NL

END

FILMED

DTIC


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

AFOSR-TR- 84-1202

(4)

AD-A149 817

AXIOMATIC CHARACTERIZATIONS OF CONTINUUM STRUCTURE FUNCTIONS*

Chul Kim and Laurence A. Baxter

Department of Applied Mathematics and Statistics
State University of New York at Stony Brook
Stony Brook, NY 11794, USA

*Research supported by the National Science Foundation under grant ECS-8306871 and by the Air Force Office of Scientific Research, AFSC, USAF, under grant AFOSR-84-0243. The US Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 84 - 1202
6a. NAME OF PERFORMING ORGANIZATION State University of New York 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) Department of Applied Mathematics and Statistics, Stony Brook NY 11794-3600	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-84-0243
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 61102F PROJECT NO.: 2304 TASK NO.: A5 WORK UNIT NO.:

11. TITLE (Include Security Classification)
AXIOMATIC CHARACTERIZATIONS OF CONTINUUM STRUCTURE FUNCTIONS

12. PERSONAL AUTHOR(S)
Chul Kim and Laurence A. Baxter

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Technical	FROM ____ TO ____	NOV 84	10

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Reliability; continuum structure function; multistate structure function.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A continuum structure function is a nondecreasing mapping from the unit hypercube to the unit interval. Axiomatic characterizations of the continuum structure functions based on the Barlow-Wu and Natvig multistate structure functions are derived.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL MAJ Brian W. Woodruff	22b. TELEPHONE NUMBER (Include Area Code) (202) 767- 5027 22c. OFFICE SYMBOL NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

ABSTRACT

A continuum structure function is a nondecreasing mapping from the unit hypercube to the unit interval. Axiomatic characterizations of the continuum structure functions based on the Barlow-Wu and Natvig multistate structure functions are derived.

KEYWORDS: Reliability; continuum structure function; multistate structure function.

AMS 1980 Subject Classification: Primary 90B25
OR/MS Index 1978 Subject Classification: Primary 721 Reliability

1. INTRODUCTION

Let $C = \{1,2,\ldots,n\}$ denote a set of components and let $\Delta = [0,1]^n$. A nondecreasing mapping $\gamma: \Delta \mapsto [0,1]$ with $\gamma(\underset{\sim}{0}) = 0$ and $\gamma(\underset{\sim}{1}) = 1$ is said to be a continuum structure function (CSF). If $\sup_{\underset{\sim}{X}\in\Delta} [\gamma(1_i,\underset{\sim}{X})-\gamma(0_i,\underset{\sim}{X})] > 0$ for each $i \in C$, where $(\delta_i,\underset{\sim}{X})$ denotes $(X_1,\ldots,X_{i-1},\delta,X_{i+1},\ldots,X_n)$, γ is said to be weakly coherent.

Definition

Let $P_1,\ldots,P_r$ denote the r minimal path sets of a binary coherent structure function. If

$$\gamma(\underset{\sim}{X}) = \max_{1\le j\le r} \min_{i\in P_j} X_i \qquad (\underset{\sim}{X}\in\Delta),$$

γ is said to be a Barlow-Wu CSF [2].

Definition

Let $\{\phi_\alpha, 0<\alpha\le 1\}$ be a class of binary coherent structure functions such that $\phi_\alpha(\underset{\sim}{Y}_\alpha)$ is a left-continuous and non-increasing function of α for fixed $\underset{\sim}{X}$ where $Y_{\alpha i}$ is the indicator of $\{X_i \ge \alpha\}$, $i=1,2,\ldots,n$. If

$$\gamma(\underset{\sim}{X}) \ge \alpha \text{ iff } \phi_\alpha(\underset{\sim}{Y}_\alpha) = 1 \qquad (\underset{\sim}{X}\in\Delta, 0<\alpha\le 1),$$

γ is said to be a Natvig CSF [3].

In this paper, we present axiomatic characterizations of the Barlow-Wu and Natvig CSFs. In particular, we show that γ is a Barlow-Wu CSF if and only if it satisfies the following conditions:

C1 γ is continuous

C2 $P_\alpha \neq \emptyset$ and $P_\alpha \subset \{0,\alpha\}^n$, $0 < \alpha \leq 1$

C3 There is no nonempty open set $A \subset \Delta$ such that γ is constant on A

C4 γ is weakly coherent

where $P_\alpha = \{\underset{\sim}{X}\in\Delta | \gamma(\underset{\sim}{X}) \geq \alpha$ whereas $\gamma(\underset{\sim}{Y}) < \alpha$ for all $\underset{\sim}{Y}<\underset{\sim}{X}\}$ and where $\underset{\sim}{Y} < \underset{\sim}{X}$ means that $\underset{\sim}{Y} \leq \underset{\sim}{X}$ but that $\underset{\sim}{Y} \neq \underset{\sim}{X}$.

Some consequences of these axioms are deduced in Section 2, and in Section 3 we present our main results: an axiomatic characterization of the Barlow-Wu CSF and an analogous characterization of the Natvig CSF. Our approach was suggested by the Borges-Rodrigues characterizations of the Barlow-Wu and Natvig multistate structure functions [5] though, as we show in Section 4, their characterizations are incorrect.

2. SOME DEDUCTIONS FROM THE AXIOMS

Let $U_\alpha = \{\underset{\sim}{X}\in\Delta | \gamma(\underset{\sim}{X}) \geq \alpha\}$ and $L_\alpha = \{\underset{\sim}{X}\in\Delta | \gamma(\underset{\sim}{X}) \leq \alpha\}$, $0 \leq \alpha \leq 1$. Further, define $K_\alpha = \{\underset{\sim}{X}\in\Delta | \gamma(\underset{\sim}{X}) \leq \alpha$ whereas $\gamma(\underset{\sim}{Y}) > \alpha$ for all $\underset{\sim}{Y} > \underset{\sim}{X}\}$, $0 \leq \alpha < 1$.

Proposition 2.1

Let γ be a CSF.

(i) γ is right (left)-continuous if and only if each $U_\alpha(L_\alpha)$ is closed.

(ii) If γ is right (left)-continuous, then each $P_\alpha(K_\alpha)$ is nonempty and $\underset{\sim}{X} \in U_\alpha(L_\alpha)$ if and only if $\underset{\sim}{X} \geq (\leq)\ \underset{\sim}{Y} \in P_\alpha(K_\alpha)$.

(iii) If γ is continuous, then $\gamma(P_\alpha) = \{\alpha\}$, $0 < \alpha \leq 1$, and $\gamma(K_\alpha) = \{\alpha\}$, $0 \leq \alpha < 1$.

Proof: The proofs of (i) and (iii) are straightforward; see [4] for the proof of (ii).

Proposition 2.2

If γ is a continuous CSF, conditions C2 and

C2' $\quad K_\alpha \neq \emptyset$ and $K_\alpha \subset \{\alpha,1\}^n$, $0 \leq \alpha < 1$

are equivalent.

Proof: Since γ is continuous, each K_α is nonempty. We show that, if C2 holds, then $K_\alpha \subset \{\alpha,1\}^n$ for all $\alpha \in [0,1)$.

Suppose, conversely, that for some $\alpha \in [0,1)$ there exists a vector $\underset{\sim}{Y} \in K_\alpha$ such that $\underset{\sim}{Y} \notin \{\alpha,1\}^n$. Then there exists at least one component, k say, such that $Y_k \notin \{\alpha,1\}$. Either $0 \leq Y_k < \alpha < 1$ or $0 \leq \alpha < Y_k < 1$; we consider these two cases separately.

Suppose, firstly, that $0 \leq Y_k < \alpha < 1$. By Proposition 2.1, $\gamma(\underset{\sim}{Y}) = \alpha$ and $\gamma(\delta_k,\underset{\sim}{Y}) > \alpha$ if $Y_k < \delta < \alpha$. Let $\gamma(\delta_k,\underset{\sim}{Y}) = \xi$; then $(\delta_k,\underset{\sim}{Y}) \in U_\xi$. Since U_ξ is closed there exists, by Proposition 2.1, an $\underset{\sim}{X} \leq (\delta_k,\underset{\sim}{Y})$ such that $\underset{\sim}{X} \in P_\xi$. Now $\underset{\sim}{Y} \notin U_\xi$ and so $Y_k < X_k \leq \delta$. Thus $0 \leq Y_k < X_k \leq \delta < \alpha < \xi$ and so $\underset{\sim}{X} \notin \{0,\xi\}^n$, in contradiction to C2.

Suppose, now, that $0 \leq \alpha < Y_k < 1$. Again $\gamma(\underset{\sim}{Y}) = \alpha$. Let $\gamma(1_k,\underset{\sim}{Y}) = \delta > \alpha$. Since $\gamma(x_k,\underset{\sim}{Y})$ is a continuous, nondecreasing function of x for fixed $(\cdot_k,\underset{\sim}{Y})$, it follows from the intermediate value theorem that, for given ξ with $\alpha < \xi < Y_k \wedge \delta$, there exists a $w \in (Y_k,1)$ such that $\gamma(w_k,\underset{\sim}{Y}) = \xi$. Thus $(w_k,\underset{\sim}{Y}) \in U_\xi$ and hence there exists an $\underset{\sim}{X} \leq (w_k,\underset{\sim}{Y})$ such that $\underset{\sim}{X} \in P_\xi$. Now $\underset{\sim}{Y} \notin U_\xi$ and so $Y_k < X_k \leq w$. It follows that $0 \leq \alpha < \xi < Y_k < X_k \leq w$ and hence $\underset{\sim}{X} \notin \{0,\xi\}^n$, in contradiction to C2.

Thus, a continuous CSF satisfying C2 also satisfies C2'. A similar argument verifies the converse. $\square$

Proposition 2.3

If γ is a CSF which satisfies C1, C2 and C3, then $\gamma(\{0,\alpha\}^n) = \{0,\alpha\}$ for all $\alpha \in [0,1]$.

Proof: If $\alpha = 0$ there is nothing to prove, so suppose that, for some $\alpha \in (0,1]$, there exists a vector $\underset{\sim}{X} \in \{0,\alpha\}^n$ such that $\beta = \gamma(\underset{\sim}{X}) \notin \{0,\alpha\}$. It is easily seen that $0 < \beta < \alpha$ and that $\underset{\sim}{X} \neq \underset{\sim}{0}$ or $\underset{\sim}{\alpha}$, and hence we can write

$$X_{i_j} = \begin{cases} 0 & \text{for } j=1,2,\ldots,k \\ \alpha & \text{for } j=k+1,\ldots,n \end{cases}$$

for some k with $1 \leq k \leq n-1$.

Since $\underset{\sim}{X} \in U_\beta \cap L_\beta$, and both are closed, it follows from Proposition 2.1 that there exist a $\underset{\sim}{Z} \in P_\beta$ and a $\underset{\sim}{W} \in K_\beta$ such that $\underset{\sim}{Z} \leq \underset{\sim}{X} \leq \underset{\sim}{W}$. This ordering will only hold if $\underset{\sim}{Z} \in \{0,\beta\}^n - \{\underset{\sim}{0}\}$ satisfies $Z_{i_j} = 0$ for $j=1,2,\ldots,k$ and if $\underset{\sim}{W} \in \{\beta,1\}^n - \{\underset{\sim}{1}\}$ satisfies $W_{i_j} = 1$ for $j=k+1,\ldots,n$ and so $A = (Z_1,W_1)\times\cdots\times(Z_n,W_n) \subset \Delta$ is open. Further, since $\underset{\sim}{Z} \in P_\beta$ and $\underset{\sim}{W} \in K_\beta$, it follows that $\gamma(\underset{\sim}{X}) = \beta$ for all $\underset{\sim}{X} \in A$, in contradiction to C3. Thus $\gamma(\underset{\sim}{X}) \in \{0,\alpha\}$ as claimed. $\square$

Proposition 2.4

If γ is a CSF which satisfies C1, C2 and C3, then $P_\alpha = \alpha P_1$ for all $\alpha \in (0,1]$.

Proof: Suppose that $\alpha < 1$, otherwise there is nothing to prove, and let $\underset{\sim}{X} \in P_\alpha$ so that $\gamma(\underset{\sim}{X}) = \alpha$. Then $\underset{\sim}{X} < \frac{1}{\alpha}\underset{\sim}{X}$ and so $\gamma(\underset{\sim}{X}) \leq \gamma(\frac{1}{\alpha}\underset{\sim}{X})$. Since $\frac{1}{\alpha}\underset{\sim}{X} \in \{0,1\}^n$, it follows from Proposition 2.3 that $\gamma(\frac{1}{\alpha}\underset{\sim}{X}) = 1$. We claim that $\frac{1}{\alpha}\underset{\sim}{X} \in P_1$.

Suppose, conversely, that $\frac{1}{\alpha}\underset{\sim}{X} \notin P_1$. Since U_1 is closed, it follows from Proposition 2.1 that there exists a $\underset{\sim}{W} < \frac{1}{\alpha}\underset{\sim}{X}$ such that $\underset{\sim}{W} \in P_1$. Consider the vector $\alpha\underset{\sim}{W} \in \{0,\alpha\}^n$; it is easily seen that $\gamma(\alpha\underset{\sim}{W}) = \alpha$ and thus there exists a vector $\alpha\underset{\sim}{W} < \underset{\sim}{X}$ such that $\alpha\underset{\sim}{W} \in U_\alpha$. This contradicts the assumption that $\underset{\sim}{X} \in P_\alpha$ and hence $\frac{1}{\alpha}\underset{\sim}{X} \in P_1$ as claimed. This holds for all $\underset{\sim}{X} \in P_\alpha$ and so $P_\alpha \subset \alpha P_1$.

Similarly, it can be shown that $\alpha P_1 \subset P_\alpha$. $\square$

3. THE CHARACTERIZATION THEOREMS

Theorem 3.1

A CSF γ is of the Barlow-Wu type if and only if it satisfies conditions C1, C2, C3 and C4.

Proof: It is easily verified that the Barlow-Wu CSF satisfies C1, C2, C3 and C4. To prove the converse, observe that

$$\begin{aligned}\gamma(\underset{\sim}{X}) \geq \alpha &\Longleftrightarrow \underset{\sim}{X} \geq \underset{\sim}{Y} \in P_\alpha \\ &\Longleftrightarrow \min_{\{i \mid Y_i = \alpha\}} X_i \geq \alpha \text{ for some } \underset{\sim}{Y} \in P_\alpha \\ &\Longleftrightarrow \max_{\underset{\sim}{Y} \in P_\alpha} \min_{\{i \mid Y_i = \alpha\}} X_i \geq \alpha\end{aligned}$$

$$\Longleftrightarrow \max_{\underset{\sim}{Y}\in\alpha P_1} \min_{\{i|Y_i=\alpha\}} X_i \geq \alpha \text{ by Proposition 2.4}$$

$$\Longleftrightarrow \max_{\underset{\sim}{Z}\in P_1} \min_{\{i|Z_i=1\}} X_i \geq \alpha \text{ where } \underset{\sim}{Z} = \frac{1}{\alpha}\underset{\sim}{Y}.$$

This holds for all $\underset{\sim}{X} \in \Delta$ and $\alpha \in (0,1]$ and so

$$\gamma(\underset{\sim}{X}) = \max_{\underset{\sim}{Z}\in P_1} \min_{\{i|Z_i=1\}} X_i.$$

Write $P_1 = \{\underset{\sim}{X}^{(1)},\ldots,\underset{\sim}{X}^{(N)}\}$ and let $T_j = \{i\in C | X_i^{(j)} = 1\}$. By the definition of P_1, it is clear that each T_j is nonempty and that $T_j \not\subset T_k$ for all $j,k=1,2,\ldots,N$ with $j \neq k$. Thus

$$\gamma(\underset{\sim}{X}) = \max_{1\leq j\leq N} \min_{i\in T_j} X_i$$

where each $T_j \subset C$. Condition C4 ensures that $\bigcup_{j=1}^{N} T_j = C$, completing the proof. $\square$

<u>Theorem 3.2</u>

A CSF γ is of the Natvig type if and only if it satisfies C2 and

C1' γ is right-continuous

C4' For each $i \in C$ and all $\alpha \in (0,1]$, there exists an $\underset{\sim}{X} \in \Delta$ such that $\gamma(\alpha_i,\underset{\sim}{X}) \geq \alpha$ whereas $\gamma(\beta_i,\underset{\sim}{X}) < \alpha$ for all $\beta < \alpha$.

<u>Proof</u>: Baxter [3] proves that Natvig CSFs are right-continuous, and it is readily seen that such functions satisfy C2 and C4'. Conversely, from the preceding proof,

$$\gamma(\underset{\sim}{X}) \geq \alpha \Longleftrightarrow \max_{\underset{\sim}{Y} \in P_\alpha} \min_{\{i|Y_i=\alpha\}} Z_{\alpha i} = 1$$

where $Z_{\alpha i}$ is the indicator of $\{X_i \geq \alpha\}$ $(0<\alpha\leq 1,\ \underset{\sim}{X} \in \Delta)$. Write $P_\alpha =$ $\{\underset{\sim}{X}^{(\alpha,1)},\ldots,\underset{\sim}{X}^{(\alpha,N(\alpha))}\}$ and let $T_j^\alpha = \{i \in C | X_i^{(\alpha,j)} = \alpha\}$, $j=1,2,\ldots,N(\alpha)$. Then $\gamma(\underset{\sim}{X}) \geq \alpha$ if and only if $\phi_\alpha(\underset{\sim}{Z}_\alpha) = 1$ where

$$\phi_\alpha(\underset{\sim}{Z}_\alpha) = \max_{1 \leq j \leq N(\alpha)} \min_{i \in T_j^\alpha} Z_{\alpha i}.$$

We claim that the binary functions $\{\phi_\alpha,\ 0<\alpha\leq 1\}$ satisfy the conditions of the definition of the Natvig CSF.

It is clear that ϕ_α is nondecreasing in each argument for all $\alpha \in (0,1]$ and that $\phi_\alpha(\underset{\sim}{Z}_\alpha)$ is nonincreasing in α for fixed $\underset{\sim}{X}$.

To verify left-continuity, it is sufficient to consider the point at which the function decreases. Thus, suppose that $\gamma(\underset{\sim}{X}) = \alpha$ $(0<\alpha<1)$; then there exists an $\underset{\sim}{X}' \leq \underset{\sim}{X}$ such that $\underset{\sim}{X}' \in P_\alpha$. Clearly, $\gamma(\underset{\sim}{X}') = \alpha$ and hence $\phi_\alpha(\underset{\sim}{Z}'_\alpha) = 1$ whereas, if $\beta > \alpha$, $\gamma(\underset{\sim}{X}') < \beta$ and so $\phi_\beta(\underset{\sim}{Z}'_\beta) = 0$. Thus $\phi_\alpha(\underset{\sim}{Z}_\alpha)$ is left-continuous as claimed.

Lastly, observe that, by C4', for each $i \in C$ and all $\alpha \in (0,1]$, there exists an $\underset{\sim}{X} \in \Delta$ such that $\phi_\alpha(1_i,\underset{\sim}{Z}_\alpha) = 1$ whereas $\phi_\alpha(0_i,\underset{\sim}{Z}_\alpha) = 0$ and so each ϕ_α is coherent.

This completes the proof. $\square$

4. SOME REMARKS ON THE BORGES-RODRIGUES CHARACTERIZATION

Let $S = \{0,1,\ldots,M\}$, $M \geq 1$. A nondecreasing mapping $\Phi: S^n \mapsto S$ with $\Phi(\underset{\sim}{0}) = 0$ and $\Phi(\underset{\sim}{M}) = M$ is said to be a multistate structure function (MSF). It is weakly coherent if $\max_{\underset{\sim}{X} \in S^n} [\Phi(M_i,\underset{\sim}{X}) - \Phi(0_i,\underset{\sim}{X})] \geq 1$ for each $i \in C$. If

$$\Phi(\underset{\sim}{X}) = \max_{1 \leq j \leq r} \min_{i \in P_j} X_i \quad (\underset{\sim}{X} \in S^n)$$

where $P_1,\ldots,P_r$ are the r minimal path sets of a binary coherent structure function, then Φ is said to be a Barlow-Wu MSF [1]. If $\Phi(\underset{\sim}{X}) \geq j$ if and only if $\phi_j(\underset{\sim}{Y}_j) = 1$ ($\underset{\sim}{X} \in S^n$, $j=1,2,\ldots,M$) where $\{\phi_1,\ldots,\phi_M\}$ is a collection of binary coherent structure functions such that $\phi_j(\underset{\sim}{Y}_j)$ is nonincreasing in j for fixed $\underset{\sim}{X}$, and where Y_{ji} is the indicator of $\{X_i \geq j\}$, then Φ is said to be a Natvig MSF [6].

Borges and Rodrigues [5] present axiomatic characterizations of the Barlow-Wu and Natvig MSFs in terms of the following conditions:

B1 For every $\underset{\sim}{X} \in S^n$ with $\Phi(\underset{\sim}{X}) \geq k \geq 1$, there exists a $\underset{\sim}{Y} \in \{0,k\}^n$ such that $\underset{\sim}{Y} \leq \underset{\sim}{X}$ and $\Phi(\underset{\sim}{Y}) \geq k$

B2 $\Phi(\{0,M\}^n) = \{0,M\}$

B3 Φ is weakly coherent.

Borges and Rodrigues [5] claim

(1) Φ is a Barlow-Wu MSF if and only if it satisfies B1, B2 and B3

(2) Φ is a Natvig MSF if and only if it satisfies B1 and B3.

Both claims are false as the following examples attest.

Example 4.1

Consider the MSF Φ_1: $\{0,1,2\}^2 \mapsto \{0,1,2\}$ defined as follows:

$$\Phi_1(0,0) = 0 \quad \Phi_1(0,1) = 0 \quad \Phi_1(0,2) = 2$$
$$\Phi_1(1,0) = 0 \quad \Phi_1(1,1) = 1 \quad \Phi_1(1,2) = 2$$
$$\Phi_1(2,0) = 2 \quad \Phi_1(2,1) = 2 \quad \Phi_1(2,2) = 2.$$

This satisfies B1, B2 and B3 and yet is clearly not of the Barlow-Wu type since the only Barlow-Wu MSFs of size two are $X_1 \wedge X_2$ and $X_1 \vee X_2$. Notice in particular that Φ_1 provides a counter-example to Lemma 4 of [5].

Example 4.2

Let $\phi_1(Y_{11},Y_{12}) = Y_{11}$ and $\phi_2(Y_{21},Y_{22}) = Y_{21} \wedge Y_{22}$ and define the MSF Φ_2: $\{0,1,2\}^2 \mapsto \{0,1,2\}$ as the function which satisfies $\Phi_2(X_1,X_2) \geq j$ if and only if $\phi_j(Y_{j1},Y_{j2}) = 1$ where Y_{ji} is the indicator of $\{X_i \geq j\}$ $(i,j,=1,2)$. This is clearly not a Natvig MSF since the binary function ϕ_1 is not coherent, but it is easily verified that Φ_2 satisfies B1 and B3.

REFERENCES

[1] Barlow, R. E. and Wu, A. S. (1978). "Coherent Systems with Multi-Components", Math. Operat. Res., 3, 275-281.

[2] Baxter, L. A. (1984). "Continuum Structures I", J. Appl. Prob., 2 (to appear).

[3] Baxter, L. A. (1984). "Continuum Structures II", submitted for pu

[4] Block, H. W. and Savits, T. H. (1984). "Continuous Multistate Structure Functions", Operat. Res., 32, 703-714.

[5] Borges, W. de S. and Rodrigues, F. W. (1983). "An Axiomatic Characterization of Multistate Coherent Structures", Math. Operat. Res., 8, 435-438.

[6] Natvig, B. (1982). "Two Suggestions of How to Define a Multistate Coherent System", Adv. Appl. Prob., 14, 434-455.

END

FILMED

3-85

DTIC